

## Finite element method modelling of the properties of a Cu–SiC composite under cyclic loading conditions

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The paper reports on finite element method (FEM) analysis of Cu–SiC composites behaviour under cyclic loading conditions. In order to emphasise the influence of materials description on following results, there were two hardening rules used to describe the plastic behaviour of the matrix: (a) simple isotropic and (b) combined isotropic-kinematic. Reinforcing ceramics was assumed to be perfectly elastic. The analysis was carried out for Cu–SiC fibre–reinforced lamina composite. The fibre volume fraction was assumed to be 20%. The modelling was based on representative volume element (RVE) geometry. Additionally, the effect of thermal residual stresses was taken into account and its influence upon the Cu–SiC composite mechanical behaviour was clarified.

Key words: *finite element method; Cu–SiC composite; plastic behaviour; RVE geometry*

### 1. Introduction

Metal matrix composites (MMCs) are high-performance materials which are used for various applications demanding high strength but low weight. This feature of MMC is widely employed in automotive and aircraft industry [1]. In recent years, there has been a significant interest in copper metal matrix composites (CuMMCs) due to their high thermal conductivity and anticipated higher operating temperature which could be higher than in the case of high strength copper alloys (CuCrZr). Due to a good creep resistance and high strength at elevated temperatures, the most promising composites are fibrous CuMMC [2, 3]. This type of composites could replace CuCrZr alloys which cannot operate at temperatures exceeding 350 °C [4–6]. CuMMCs may be applied in particular elements of supersonic speed vehicles, like rockets and aircrafts, where they can be used as the attack edges of wings and the facing material in combustion chambers. It is also important that copper has a high resistance to hydro-

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gen embrittlement as hydrogen is likely to be the predominant fuel. Materials with high thermal conductivity are indispensable in order to reduce the temperature gradient in the high heat flux areas of engines. CuMMCs are also candidate materials for the next generation thermonuclear reactors [7, 8]. Recently CuSiC MMCs have been considered as heat sink materials for fusion energy applications. The most promising reinforcement material for a Cu matrix is silicon carbide fibre SCS-6 type produced by Speciality Materials (Fig. 1). It can be used to produce lamina or laminate composites of various fibre volume fractions and gives an opportunity to manufacture gradient materials, which is particularly important for the joining technology.

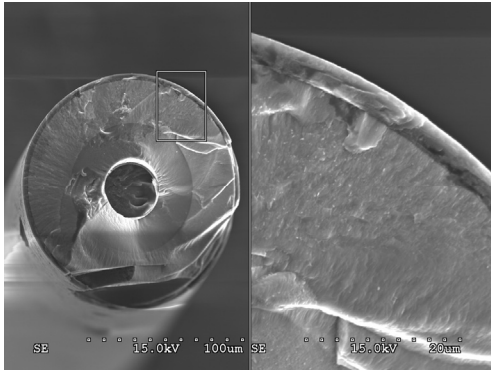


Fig. 1. SEM image of SCS-6 fibre

Due to the variation of temperature and external load applied to MMCs components, MMCs are usually subject to cyclic loading, which could lead to a quick component failure, especially if plastic flow occurs. In the case of CuMMC, plastic deformation of the Cu matrix can take place easily and more frequently than in other metal matrices. One can expect easy failure by means of low cyclic fatigue and debonding at the interface, leading to a deterioration of composite properties.

Internal stresses and residual deformation can be induced in MMC by the differences in the thermal expansion coefficients (CTE), usually small for ceramic fibres and much higher for metals matrices. Also, temperature fluctuation during the service of CuMMCs may lead to accumulation of plastic deformation that could be: (a) regressive (shakedown), (b) constant (plastic shakedown) or (c) progressive (ratcheting), depending on the load amplitude level, the microstructure and the properties of the components. Generally, MMCs are produced using high temperature techniques which create residual stresses in the composite during the cooling process. The magnitude and distribution of these stresses depend on the micro-geometry, properties of the components, and the manufacturing conditions [9].

Currently, step-by-step simulations based on well developed non-linear finite element method (FEM) solutions are efficient and sufficiently accurate to predict the properties of composites, including the residual stresses and loading history. These methods can also be used to simulate the lifetime of a composite under cyclic loading.

In this study, a unit cell approach was employed to predict the mechanical behaviour of CuMMC under various conditions of thermo-mechanical loadings. Simplified isotropic hardening and more advanced combined hardening plasticity models were used for the simulation of the stress-strain response of the Cu matrix. SiC fibres were considered to be perfectly adherent to the Cu matrix and no debonding effects at the interface were taken in to account.

### 1.1. Plastic deformation rules

The main concept of plastic deformation and hardening rules is based on the yield surface which is embedded in the stress domain. The shape of this surface depends on the behaviour of material. The plasticity surface is always convex. This is sufficient to determine the material's "reaction" during an arbitrary loading and unloading path. For most metals it has a cylindrical shape, axially symmetrical around a stress tensor direction describing pure hydrostatic stress state:

$$\sigma_H = \frac{\sigma_{ii}}{3}, \quad i = 1, 2, 3 \quad \text{for a principal stress} \quad (1)$$

When the stress tensor reaches the surface and when the product of the strain increment and stress increment is higher than/or equal zero, a plastic flow is possible. Plastic deformation is controlled by the stress and yield surface in accordance with an associated plastic flow rule described below:

$$\partial \varepsilon_{pl} = \frac{\partial F(\sigma_{ij})}{\partial \sigma_{ij}} \quad (2)$$

where  $F(\sigma_{ij})$  is the plastic potential associated with plastic surface [10];

$$F(\sigma_{ij}) = \sqrt{3} \frac{1}{2} \sqrt{S_{ij} S_{ij}} \quad (3)$$

where  $S_{ij}$  is the deviatoric stress tensor.

In the case of plastic deformation, the yield surface may change its shape, size, or both. A change of the yield surface size is connected to isotropic hardening and change of location of the centre (known as kinematic hardening). The transformation of the yield surface is related to material hardening or softening.

### 1.2. Isotropic hardening

The basic rule describing the behaviour in the plastic domain is the isotropic hardening. Isotropic hardening describes a uniform expansion of the yield surface (in the

deviatoric plane). It implies that the position of the centre of the yielding surface remains in the same position during plastic flow [11]. This property leads to the increase of yield stress irrespective of the loading direction. For isotropic hardening it is not important under what conditions (stress tensor structure) the yielding surface was reached (Fig. 2).

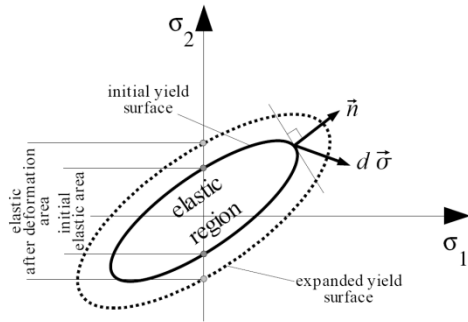


Fig. 2. Schematic evolution of isotropic hardening model

Many materials exhibit such a type of plastic behaviour. Purely isotropic hardening provides elastic shakedown in the first cycles and is generally used for modelling under unidirectional monotonic loading. In many cases, kinematic hardening is neglected. For shakedown – ratcheting analysis, the kinematic rule plays a more significant role where the prediction of behaviour under cyclic loading is required [12]:

$$F(\sigma, H) = 0 \quad (4)$$

$$H = h(\sigma_0) \quad (5)$$

$$\sigma_0 = \sigma_{s0} + Q_\infty (1 - e^{-b\varepsilon_{pl}}) \quad (6)$$

Equation (6) describes the development of yield surface as a non-linear function of plastic deformation where  $\varepsilon_{pl}$  is the equivalent plastic strain.  $Q_\infty$  indicates the maximum change in the size of yield surface and is the rate at which the yield surface develops as  $\varepsilon_{pl}$  increases.

### 1.3. Kinematic hardening

Kinematic hardening takes place when the back stress tensor  $\alpha$  translates the yield surface centre to a new position (Fig. 3). This rule was developed to model the Buschinger effect [13], where the yield stress is higher when the material is loaded in one direction and gets smaller for flowed reverse loading case. The back stress tensor depends on the plastic strain rate. This dependence between  $\alpha$  and  $d\varepsilon/dt$  could be simply linear or non-linear.

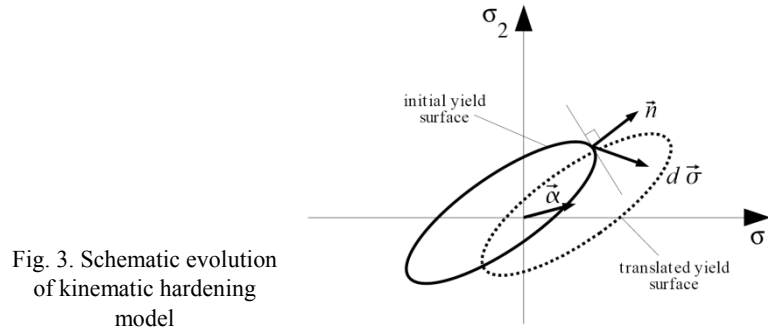


Fig. 3. Schematic evolution of kinematic hardening model

### 1.4. Combined model

The combined model is based on isotropic and kinematic systems interacting with one another which causes that the expansion and translation of the yield surface simultaneously take place. In this case, it is hard to distinguish which one dominates. The balance between them is set by the parameters of governing equations. The parameters can be determined only when the effects of both models can be separated. For example, it is possible, taking into account the unidirectional cyclic test, when the stress-strain loop is saturated and only kinematic hardening is active since the elastic domain has reached its maximum value  $Q_\infty$  [14].

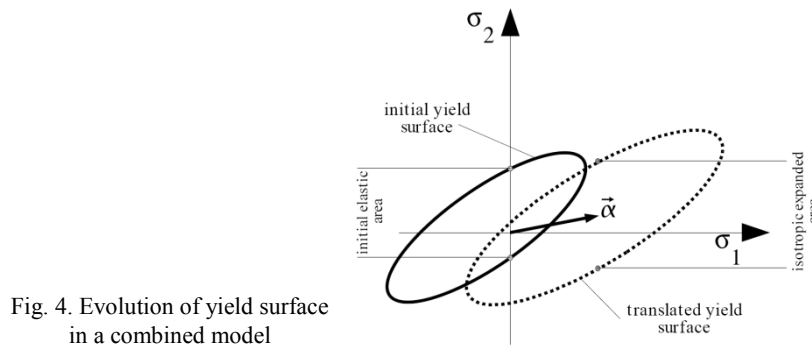


Fig. 4. Evolution of yield surface in a combined model

## 2. Results

### 2.1. Unit cell model (UCM)

A composite structure is frequently periodic, which simplifies micro-modelling. For the entire volume of a composite, assuming a UCM, it is possible to define a rep-

representative volume element (RVE) as the subject of simulation. The unit cell model is commonly used in the FEM to calculate the macroscopic properties of a material [15].

The UCM provides information on the micro-mechanical as well as the macroscopic properties of composites. For structural analysis, the UCM could deliver data for further homogenization. This approach is very effective and allows one to reduce the calculation time but in many cases requires developing new constitutive models.

## 2.2. Composite

The investigated material was a copper matrix composite containing embedded SCS-6 SiC fibres (Fig. 1). The fibre volume fraction was 20%. It was assumed that the fibres were well bonded and no interface cracking was taken into account. Because of the high strength of the SiC fibres, (Fig. 5, [16]) they were considered to be perfectly elastic. The Cu matrix exhibited the elastic-plastic behaviour (Fig. 6). Two constitutive models have been taken into account to describe the properties of copper: a) purely isotropic non-linear, b) combined non-linear isotropic-kinematic hardening, based on the Armstrong and Frederick rule, under the Lemaitre and Chaboche framework [17]. The parameters for the models were calibrated, based on experimental results obtained from cyclic tests.

The strain-controlled experiments were carried out using axial tension-compression unit made by Instron. The specimen deformations were measured by an extensometer. The stabilization of the stress-strain loop was considered to appear after 50 cycles, above this number there were no considerable changes of its shape.

The UCM contained SiC fibres 144  $\mu\text{m}$  in diameter surrounded by the Cu matrix. Periodic boundary conditions were employed to simulate an ideal bulk composite without any influence of free surfaces (Fig. 7).

By introducing 3D geometry, it was possible to fully cover the longitudinal, as well as the transverse, response of the composite. The finite element (FE) model was assembled with C3D8R 3D solid elements of reduced integration. The overall dimensions of the model were  $143 \times 143 \mu\text{m}^2$  in the transverse plane and the thickness of 10  $\mu\text{m}$ .

Several numerical analyses were carried out using the commercial code ABAQUS. Calculations were performed for uniaxial straining and the macro stress-strain curves were modelled.

The residual stresses were introduced by simulating the fabrication processes. The composite was “numerically” annealed at 1000 °C and then cooled to room temperature. The material properties were calibrated from cyclic test experimental data. Calculations for the stress free state of the composite (without residual stresses) were carried out for comparison.

Monotonic stress-strain behaviour of the matrix material, for the isotropic model and the combined one, are presented in Fig. 9. It is shown that both strain-stress curves are initially the same.

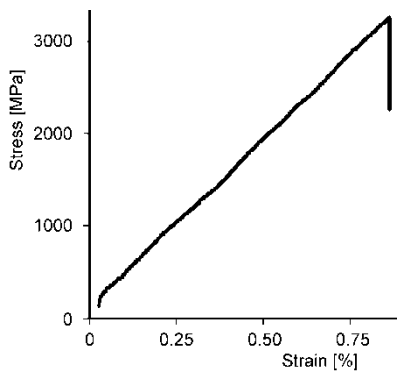


Fig. 5. Typical stress-strain diagram for SiC fibres

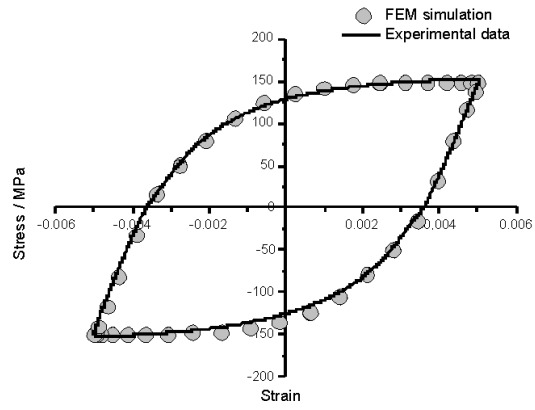


Fig. 6. Cu stress-strain curve for a saturated cycle

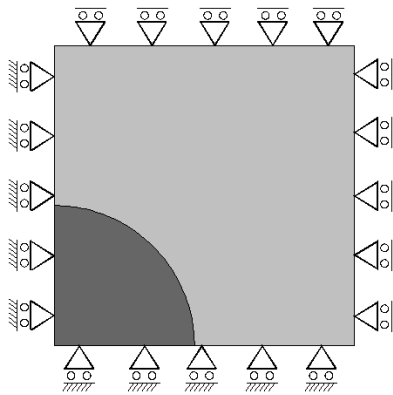


Fig. 7. Composite UCM constraints

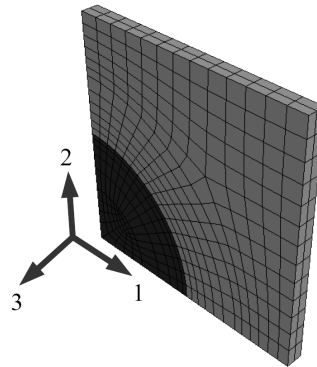
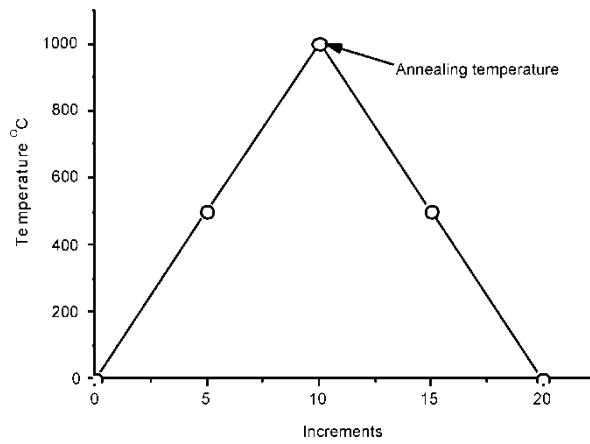


Fig. 8. Unit cell model of lamina (20% volume fraction of SiC fibres) submitted for calculations

Fig. 9. Composite heating-cooling step conditions



In the case of annealed copper, the experimental results as well as literature data [18, 19] show a strong Bauschinger effect which is more critical if reversible loading occurs in the issue, e.g., heating–cooling. During numerical analysis, the macroscopic strain–stress response of composite was obtained. The values were calculated as follows:

Macroscopic stress is defined as an integral

$$\Sigma_{ij} = \frac{\int \sigma_{ij} dV}{V} \quad (7)$$

For discrete FE volume it becomes

$$\Sigma_{ij} = \frac{\sum_1^n \sigma_{ij} V_n}{V} \quad (8)$$

where  $n$  is the number of finite elements and  $V_n$  is the volume of the  $n$ -th element.

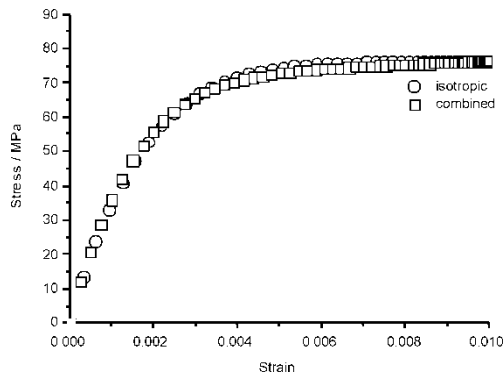


Fig. 10. Stress–strain response of pure copper (matrix material) calculated for various constitutive models (combined non-linear isotropic-kinematic hardening and non-linear isotropic hardening)

The strains are calculated in an analogous way:

$$E_{ij} = \frac{\sum_1^n \varepsilon_{ij} V_n}{V} \quad (9)$$

### 3. Discussion

The materials used for plasma reactor applications, especially the heat sink components, are supposed to serve under cyclic loading conditions [8]. Such conditions may cause damage to the structure, especially when the difference in CTE of the com-

ponents is high and one of the components exhibits tendency for plastic deformation. However, it could be accommodated by shakedown, while progressive hardening of components causes the return of the structure to elastic regime. The aim of this work is to demonstrate that for prediction of the shakedown effect and stress limits for composite materials, realistic models are needed instead of the pure isotropic hardening model, commonly used in engineering practice. However, the simulation shows that the residual stresses in Cu–SiC do not affect strongly the composite response under monotonic loading for both material models (Figs. 11–13).

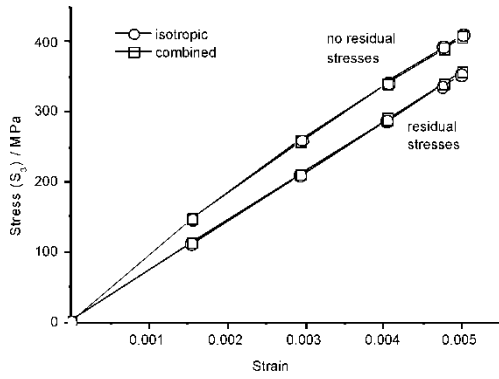


Fig. 11. Calculated macroscopic stress–global strain curves for composite axial loading case parallel to direction of fibres (with and without residual stresses)

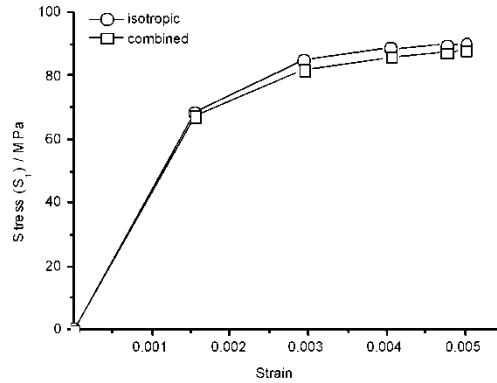


Fig. 12. Composite stress–strain curves for transverse loading case (perpendicular to direction of fibres) without residual stresses

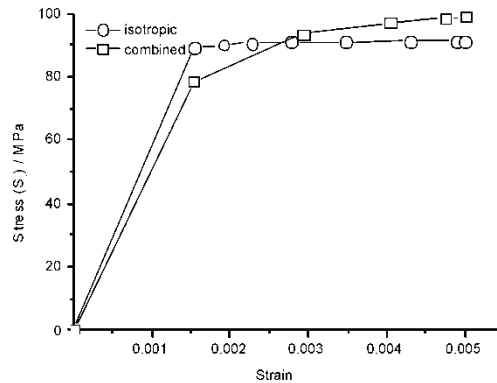


Fig. 13. Composite stress–strain curves for transverse loading case (perpendicular to fibres direction) with residual stresses

In this paper, some insight into the cyclic properties has been obtained. Calculations have been performed for strain- and stress-controlled modes. The cyclic simulations have been carried out for both presented constitutive models up to 10 cycles. The coefficient of amplitude asymmetry  $R$  was set to  $-1$ , which indicates a fully reversed loading case.

For stress-controlled, as well as for the strain-controlled mode, differences appeared after the first cycles between the two considered constitutive models of plastic-

ity. For Cu–SiC composite, the applied isotropic hardening model predicts elastic shakedown in the first few cycles while for the combined model only plastic shakedown is obtained (Figs. 14, 15). This is mostly due to a strong Bauschinger effect of Cu. Thus, it could be concluded that applying the isotropic hardening model one can overestimate the accommodation of plastic deformation (shakedown). However, for the combined model, due to the low initial yield stress  $\sigma_0$ , it is difficult to determine a safe load condition in stress space, which could bring composite back to the elastic domain. In fact, plastic energy dissipated in each cycle for the applied stress exceeded yield stress. The amount of plastic energy dissipated in each cycle could be used as a parameter to determine the load limits for the composite, assuming that a limited plastic deformation is allowed.

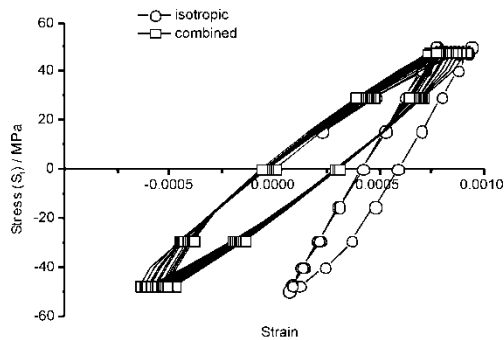


Fig. 14. Calculated global composite stress–strain curves for transverse cyclic loading case, stress controlled without residual stresses

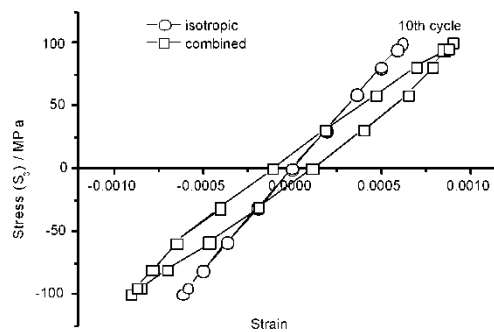


Fig. 15. Calculated global composite stress–strain curves for fibre direction cyclic loading case, stress controlled, without residual stresses

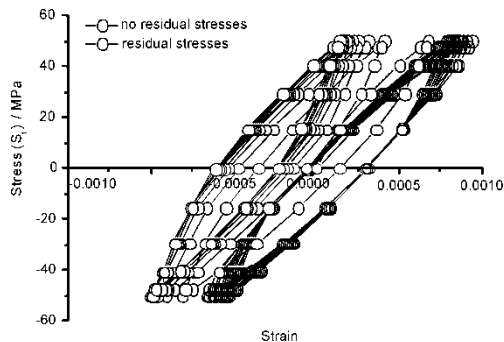


Fig. 16. Calculated global composite stress–strain curves for transverse direction cycling with and without residual stresses

## 4. Conclusions

The combined Armstrong–Frederick–Chaboche model has shown to be in good agreement with the experimental cyclic stress–strain curves. Micromechanical model-

ling of Cu–SiC composite, based on cyclic plasticity, is more relevant to predict load limits of composites under thermal and mechanical cyclic conditions, which is not covered by the non-linear isotropic model.

Residual stresses inducted during simulation of the manufacturing process do not influence further the cyclic behaviour of Cu–SiC composite and do not affect plastic energy dissipation. This is mostly due to low plastic yield of the annealed Cu.

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