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# Spin-dependent transport through a double dot system

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The coherent transport through a set of two capacitively coupled quantum dots placed in a magnetic field and coupled to ferromagnetic electrodes is considered in the limit of infinite intra- and interdot interactions. The densities of states are calculated in an approximation that favours separate fluctuations of spin, orbital isospin, and simultaneous fluctuations of both of them. Apart from the Kondo peak, satellite many-body peaks are also found in the densities of states. Their positions and weights depend on the magnetic field and polarization of electrodes. This is reflected in the spin dependence of conductance remarkably varying for the voltage bias corresponding to the peak positions.

Key words: *electronic transport; nanoscopic systems; Kondo effect*

## 1. Introduction

There is currently much interest in understanding spin-dependent electron transport in nanostructures. Spin-based devices hold promises for future applications in conventional [1] as well as quantum computer hardware [2]. Recent interest in manipulating spins in nanostructures is based on the ability to control and maintain spin coherence over practical length and time scales. In nanoscopic dots, the charging energy plays a dominant role and correlation effects are of importance. At low temperatures and strong coupling $\Gamma$ between the dot and the leads ($k_BT << \Gamma$), quantum fluctuations in the charge and spin degrees of freedom strongly affect transport through the dot. Spin fluctuations lead to the Kondo effect, which has been verified by many experiments on single dots [3]. Recently, the Kondo effect has also been observed in double dot structures [4]. The Kondo effect has two possible sources in coupled quantum dots (DQD): spin and "orbital" degeneracies. The large number of tunable parameters in DQD systems allows delicate manipulation of the Kondo physics. An

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interesting issue is how the Kondo physics is affected by the polarization of electrodes. The attachment of ferromagnetic leads to carbon nanotubes has been reported [5], and carbon–nanotube QDs have been shown to display the Kondo physics below an unusually high temperature [6]. Other possible realizations of magnetic nanostructures are spin-polarized STMs and magnetic tunnel junctions with magnetic impurities in the barrier [7, 8].

Motivated by these experiments, we discuss in the present paper a spin-dependent transport within a simple model of a double dot system (DQD). It is assumed that the dots are capacitively coupled and that magnetic electrodes are attached to them. The aim of our work is to discuss the influence of the magnetic field and polarization of the leads on the many-body structure of the density of states and on the spin dependence of conductance.

## 2. Model

We discuss a system of two capacitively coupled quantum dots placed in a magnetic field. Each dot is connected to separate pairs of ferromagnetic electrodes. The system is modelled by the two-dot Anderson Hamiltonian with a single level at each dot and an additional term for interdot interaction

$$H = \sum_{kri\sigma} \varepsilon_{kri\sigma} c^{+}_{kri\sigma} c_{kri\sigma} + \sum_{i\sigma} \varepsilon_{i\sigma} c^{+}_{i\sigma} c_{i\sigma} + \sum_{i} U_i n_{i+} n_{i-} + U \sum_{\sigma\sigma'} n_{1\sigma} n_{2\sigma'} + \sum_{kri\sigma} t_{ri} (c^{+}_{kri\sigma} c_{i\sigma} + h.c) \quad (1)$$

where $i$ numbers the dots ($i = 1, 2$), and leads being labelled $(i, r)$ $(r = L, R)$. $\varepsilon_{i\sigma} = \varepsilon_i + \sigma h$, $\sigma = \pm 1$ (we set $|e| = g = \mu_B = k_B = 1$). The first term describes electrons in the electrodes, the second represents the field-dependent site energies, the third and fourth account for intra and intercoulomb interactions, and the last one describes the tunnelling.

The ferromagnetism of the leads is accounted for by different densities of states for up and down-spin electrons. The wide-band limit is used, with the densities of states of the electrons in the leads assumed constant, $\rho_{i\sigma} = 1/2D_{i\sigma}$, where $|\varepsilon| < D_{i\sigma}$, and $D_{i\sigma}$ is half of the bandwidth.

For simplicity, we restrict ourselves to the case of identical dots ($\varepsilon_i \equiv \varepsilon_0$), identical electrodes, and equal couplings to the dots, i.e. $t_{ri} \equiv t$. The bare Green's functions of the electrodes

$$g_{r\sigma} = \sum_{k} g_{kr\sigma} = \sum_{k} \frac{1}{\omega - \varepsilon_{kr\sigma}} \equiv g$$

are taken in the form $g = -i\pi\rho_0$. Consequently, the elastic couplings to the electrodes are independent of energy: $\Gamma_{ir\sigma}(\omega) = 2\pi t^2 \rho_0 \equiv \Gamma_\sigma$. One can define the spin polarization of the electrodes as $p = (\Gamma_\uparrow - \Gamma_\downarrow)/(\Gamma_\uparrow + \Gamma_\downarrow)$.

Within the Keldysh formalism, the current in the DQD, $I = \sum_{i\sigma} I_{i\sigma}$ [9], has the form

$$I_{i\sigma} = \frac{e}{\hbar}\int d\omega \frac{\Gamma_{iL\sigma}(\omega)\Gamma_{iR\sigma}(\omega)}{\Gamma_{iL\sigma}(\omega)+\Gamma_{iR\sigma}(\omega)}[f_{iL}(\omega) - f_{iR}(\omega)]\rho_{i\sigma}(\omega) \tag{2}$$

where $\rho_{i\sigma}(\omega) = (-1/\pi)\mathrm{Im}\, G^r_{i\sigma}(\omega)$, and $f_{ir}$ are the Fermi distribution functions of the electrodes. The current, the distribution functions, and Green's functions are also functions of temperature, field, polarization, and voltage bias. Spin-resolved, non -linear conductance can be calculated from Eq. (2) by a numerical derivative $\breve{G}_\sigma(V) \equiv \dfrac{\partial I_\sigma}{\partial V}$, $I_\sigma = I_{1\sigma} + I_{2\sigma}$.

For strong interactions, $(U_1, U_2, U) \to \infty$, and a deep dot level ($-\varepsilon_{i\sigma} >> \Gamma$ ), retarded Green's function can be approximated by the following multipole expression

$$G^r_{i\sigma}(\omega) = \frac{1-n_\Omega}{3}\sum_{l\in\Omega} \frac{1}{\omega - \varepsilon_{i\sigma} - \Sigma_0 - \Sigma^1_{i\sigma,l}(\omega)} \tag{3}$$

where $\Sigma_0 = -i\Gamma$ is the self-energy for the noninteracting QD due to tunnelling of the $i\sigma$ electron, $\Omega$ is a set of quantum numbers labelling the virtual intermediate states in the tunnelling, and $n_\Omega$ denotes the average total occupation of these states. $\Omega = \{(\bar{i}\,\sigma),(i,-\sigma),(\bar{i},-\sigma)\}$, $n_\Omega = \sum_{l\in\Omega}\langle n_l\rangle$, and $\{\sum^1_{i\sigma,l}\}$ denotes the correlation parts of the self-energy ($\bar{1} = 2$, $\bar{2} = 1$). Expression (3) is a simple generalization of the single dot formula of Meier et al. [9] to the DQD system case, derived by the equation of motion (EOM) technique with the decoupling procedure for higher order Green's functions, which neglect correlations in the leads. Formulae (3) and (4) correspond to the approximation that separately takes into account isospin fluctuations, spin fluctuations, and fluctuations in strongly coupled spin and isospin

$$\sum\nolimits^1_{i\sigma,i'\sigma'} = t^2 \sum_{k\in L,R} \frac{f_{L,R}(\varepsilon_{ki'\sigma'})}{\omega - \varepsilon_{i\sigma} + \varepsilon_{i'\sigma'} - \varepsilon_{ki'\sigma'} + i\delta_{i\sigma'}} \tag{4}$$

$\delta_{i\sigma}$ describes decoherence due to a finite voltage bias or field-induced level splitting. An estimate of the lifetime can be obtained from perturbation theory [9].

The first sequence of correlated tunnelling, represented by $\sum^1_{i\sigma,\bar{i}\sigma}$, ccurs through the intermediate virtual states of the same spin but from a different dot. They induce fluctuations in single dot occupations (the orbital isospin flips). As a result, the orbital Kondo resonances are built up for each spin channel. The second type of tunnelling process, $\sum^1_{i\sigma,i\text{-}\sigma} = \sum^1_{i\sigma,\bar{i}\text{-}\sigma}$, links the non-degenerate states and causes the singularity of self-energy in regions separated from the Fermi level by Zeeman splitting. In the limit of vanishing magnetic field, they correspond to the spin Kondo effect and to simultaneous fluctuations in spin and isospin. The neglected processes which mix the above-mentioned three types of fluctuations are of special importance for systems close to full fourfold spin-orbital degeneracy. Their role is under investigation by a more careful treatment of EOM equations, and the results will be published elsewhere.

## 3. Numerical results

We present numerical results for the deep Kondo limit $\varepsilon_0 = -4\Gamma$. $\Gamma = \Gamma_\uparrow$ is taken as the energy unit. The bandwidth of the leads is taken as $D_\uparrow = 50\Gamma$. We do not discuss any decoherence effects, and the calculations were performed with $\delta = \delta_{i\sigma} = 10^{-4}$.

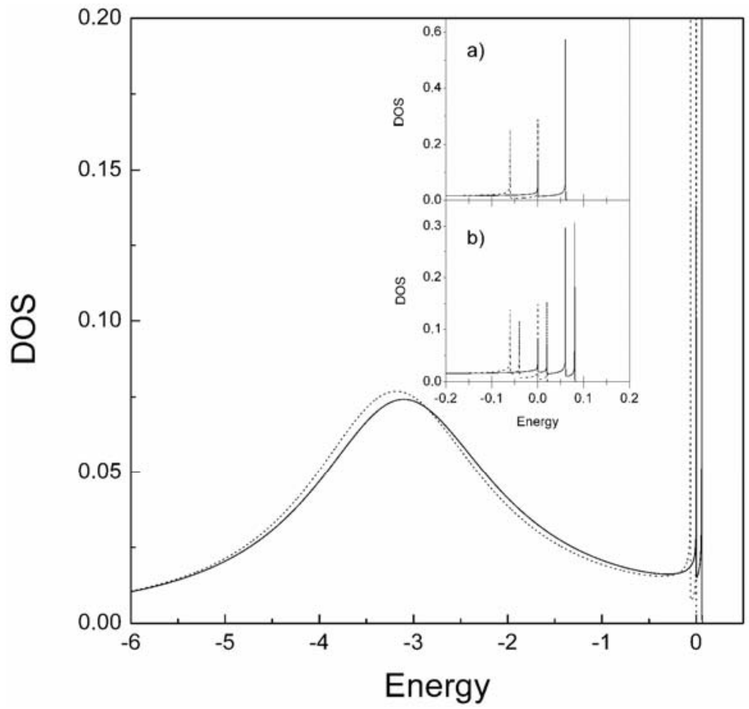


Fig. 1. Total density of states of a capacitively coupled double quantum dot in a magnetic field $h = 0.03$ and for a polarization of the leads $p = 0.05$, calculated for the bare dot energy $\varepsilon_0 = -4$. Spin up is represented by the solid line and spin down by the dotted line. Inset (a) shows the many-body structure for zero bias voltage, $V = 0$, and inset (b) for $V = 0.02$

In Figure 1, we plot the spin-dependent DOS of DQD for a finite magnetic field, a small polarization of the leads, and vanishing voltage bias. The broad charge fluctuation peak is split by the field. The widths of the spin-dependent peaks are slightly different, which is a consequence of the polarization-introduced difference in the tunnelling rates $\Gamma_\sigma$. Also, a shift of the centre of mass of the charge fluctuation peak is observed reflecting the renormalization of the dot levels by spin and orbital isospin fluctuations. The triple-peak many-body structure is seen close to the Fermi level. A similar result was reported earlier by Pohjola et al., who performed the calculations within the resonant-tunnelling approximation [10].

For clarity, we also show the curves of DOS in a narrower energy region in the inset. Both spins contribute to the central peak (orbital isospin fluctuations). The satellite peaks, located roughly in the positions $\pm 2h$, are characterized by different spin polarizations. This property should manifest itself in the spin dependence of boson -assisted tunnelling but the possible experimental observation can be masked by decoherence. Inset b) shows the many-body structure of DOS for finite $V$. The source -drain voltage causes the peaks to split.

For $V \approx \pm 2h$, the satellite peaks of opposite spin polarization enter the energy region between the Fermi levels of the leads from opposite sides, and consequently peaks in the differential conductance build up. This is illustrated in Fig. 2. The fact that the positions of the non-linear conductance peaks are determined by the strength of the field can be used for measuring the field.

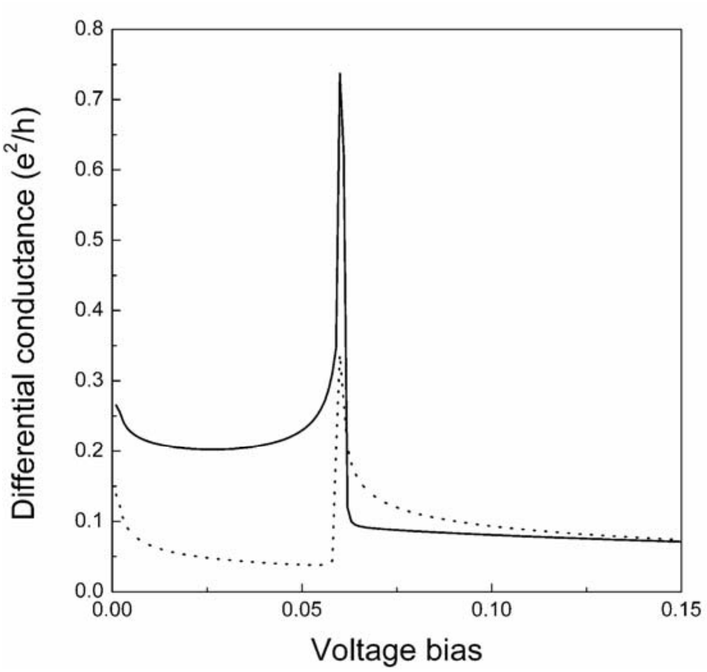


Fig. 2. Differential conductance vs. applied bias for spin up (solid line) and spin down (dotted line). A magnetic field $h = 0.03$ and an electrode polarization $p = 0.05$ were chosen

In the following pictures, we discuss the spin dependence of conductance by presenting its polarization, defined as $P_{\text{con}} \equiv (\breve{G}_\uparrow - \breve{G}_\downarrow)/(\breve{G}_\uparrow + \breve{G}_\downarrow)$. Figure 3a presents the polarization of conductance for a fixed magnetic field and three different polarizations of the leads. Similar dependencies but for a fixed polarization of the electrodes and different fields are shown in Fig. 3b.

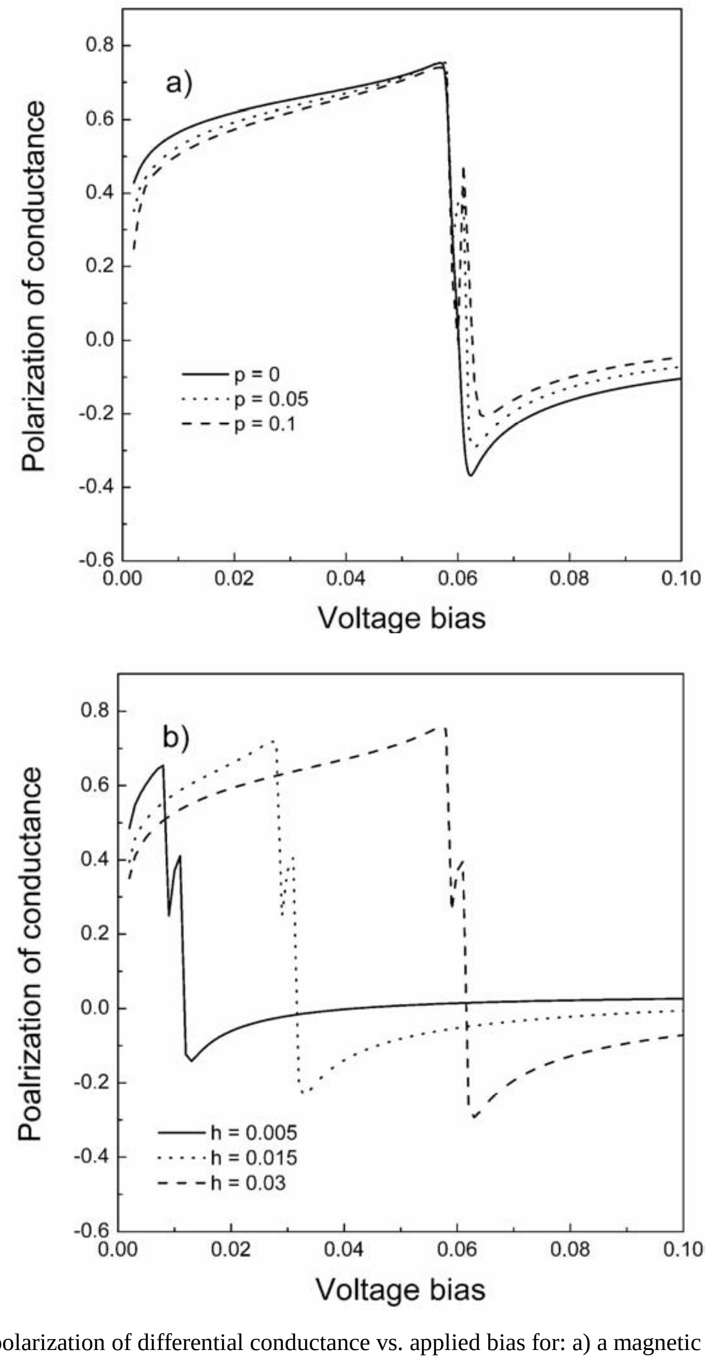


Fig. 3. The polarization of differential conductance vs. applied bias for: a) a magnetic field $h = 0.03$ and three different electrode polarizations, b) $p = 0.05$ and three different values of the magnetic field

Remarkable changes in the spin polarization of conductance are observed for voltages determined by the peak positions ($eV \approx 0$ and $eV \approx \pm 2h$). They reflect the spin asymmetry of the density of states, which increases with increasing field or polarization. Apart from the intensity differences (see, e.g., inset b) of Fig. 1), the positions of

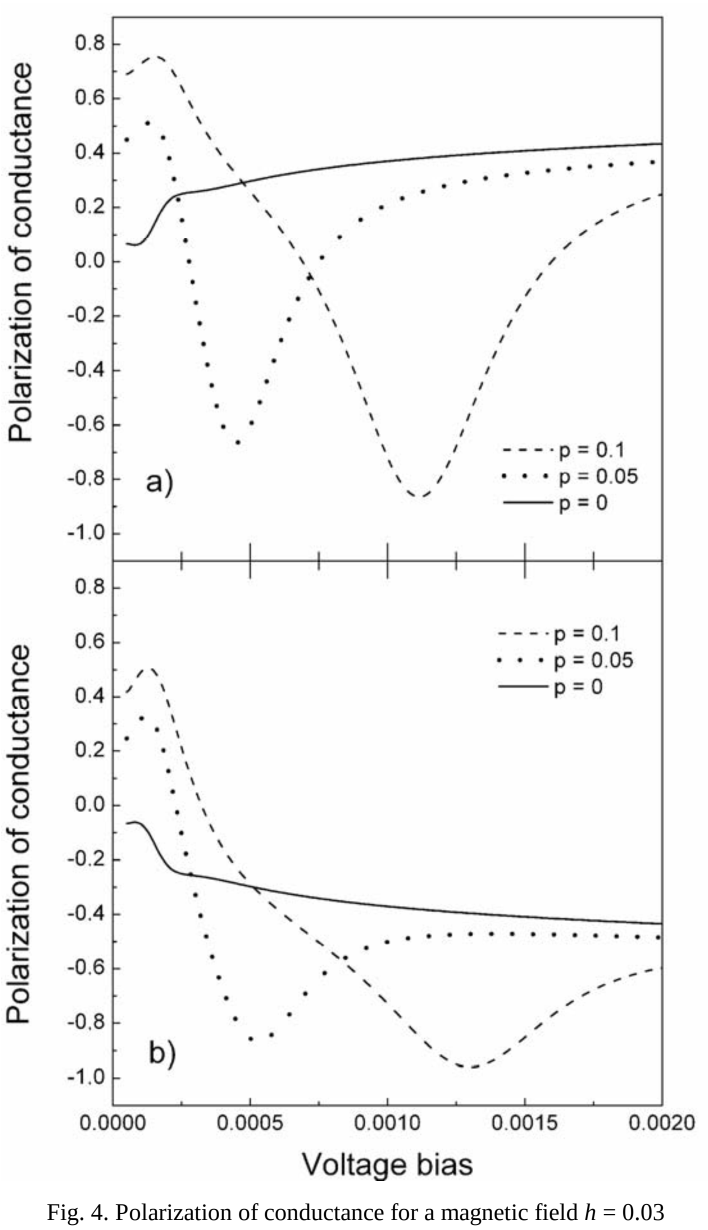


Fig. 4. Polarization of conductance for a magnetic field $h = 0.03$ and three different electrode polarizations for small bias

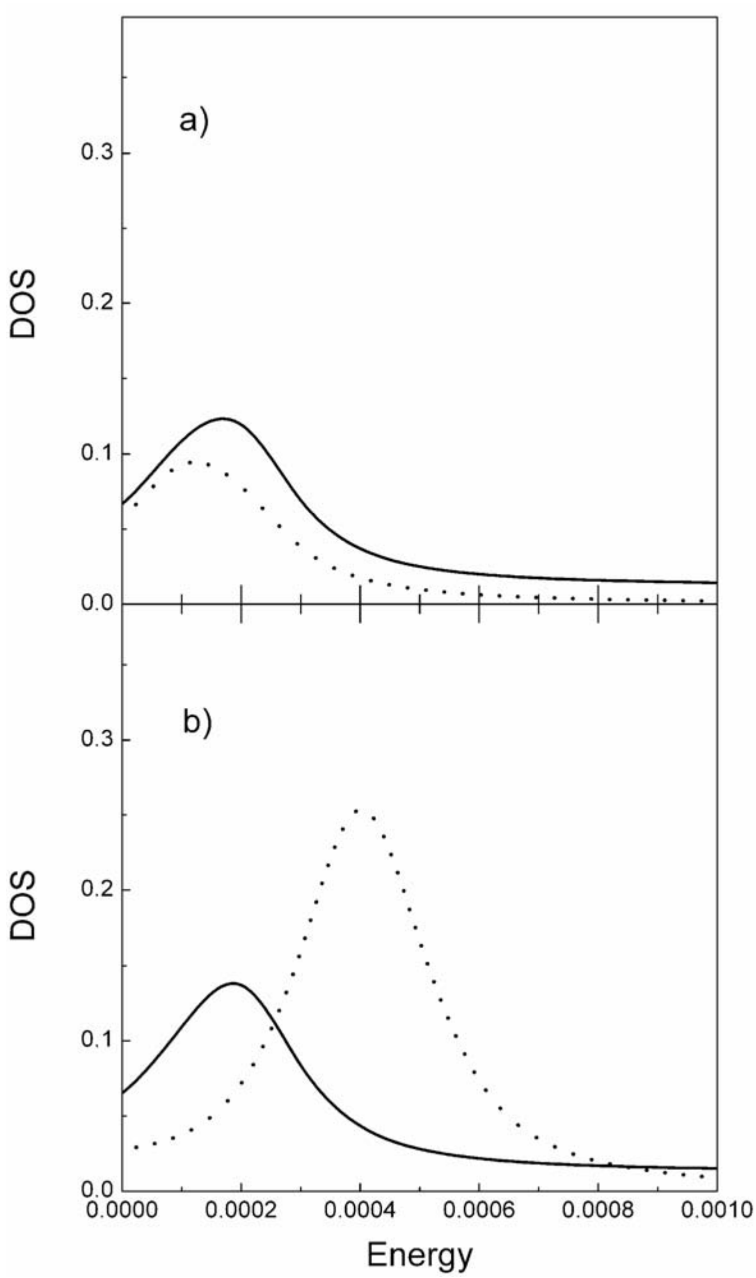


Fig. 5. Densities of states for spin up (solid line) and spin down (dotted line) for: a) $p = 0$, b) $p = 0.05$ in the narrow energy region close to the Fermi level

the corresponding peaks with opposite spin orientations also differ slightly. The latter effect is only weakly reflected in Fig. 3; its traces are seen in the narrow oscillations of some curves near the satellite peaks. The many-body peaks are very narrow and the mentioned shift of the peaks is small. The results in Fig. 3 are presented for a voltage step of $\Delta V = 10^{-3}$. To get a more detailed insight into the mentioned subtle effect, we focus in the following exclusively on the narrow voltage region $V < 2\cdot10^{-4}$, presenting these curves with the voltage steps of $\Delta V = 10^{-5}$. Figure 4a shows polarization of conductance for $h = 0.03$ for unpolarised leads ($p = 0$) and for two small values of polarizations. Figure 4b presents analogous curves but for a reversed field at the dot. For $p = 0$, the curve is a mirror reflection of the corresponding curve from Fig. 4a. For a finite polarization of the leads, the consequence of changing the relative orientation of the field and polarization in the leads from antiparallel (Fig. 4a) to parallel (Fig. 4b) is visible. To understand the presented spin dependencies, in Fig. 5 we show the evolution of the DOS with the lead polarization for the relevant energy region. The oscillations in the polarization of conductance observed for voltages close to $V = 0$ reflect the polarization-induced separation of opposite spin peaks located in the vicinity of the Fermi level. The approach adopted in this paper is rather crude, hence the results obtained, describing subtle effects, should be taken with some caution. Only a qualitative agreement can be expected, moreover, the effects mentioned can be easily masked by temperature.

## 4. Conclusions

We have studied spin-dependent electronic transport through a capacitively coupled double quantum dot placed in a magnetic field and attached to a ferromagnetic lead. The influence of the field and polarization of electrodes was examined. Apart from the central Kondo peak, two satellite peaks below and above the Fermi level are observed in the many-body structure of DOSs. The satellite peaks are characterized by different spin polarizations. The field and polarization of electrodes introduce weak asymmetry in the weights and positions of the peaks, which rises when the field or polarization is increased or when moving towards the mixed valence range. This reflects strong variation of the polarization of conductance in the vicinity of many body resonances, where even a change in the sign of polarization is observed. The purpose of this paper was to obtain only a preliminary, very crude picture of possible influence of external magnetic field on the polarization of conductance in DQDs. More advanced considerations of this problem are under way.

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