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Analysis of the wave packet interference pattern in the Young experiment

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At its beginning, the quantum mechanics has been so controversial theory that not all physicist were able to agree with its assumptions. Nowadays, it seems that the problem does not exist any more, although the quantum theory is still incomplete. The main point of the discussion, which has been raised all the time, is the problem of the measurement understood as the influence of an observer or a detector presence on the wave packet describing the state of the observed system. In this paper, the problem of the influence of the detector on the state of the system is reported on the basis of two-slit experiment described in new formalism called projection evolution. This new approach connects two ways of state evolution: unitary evolution and evolution visible during the measurement.

Key words: *quantum measurement; projection evolution; two-slit experiment*

1. Projection evolution

There are two possible ways of evolution of the quantum state of a system, which is completely predictable and reversible, called unitary evolution, as well as the irreversible rapid process connected with measurements [1]. There have been a few attempts of bringing these two possible ways of evolution together but up to now none of them has been fully satisfying.

The projection evolution is considered as a new fundamental law of quantum evolution [2]. The classical formalism of quantum mechanics has been applied for the description of that theory. The only change done in this formalism is that the unitary time evolution and the projection postulate were replaced by the projection evolution. In such an approach, the evolution is simply considered as a sequence of measurements made by Nature occurring with a specified probability determined by the state of the system. The new theory is concise an transforms into a unitary evolution and the theory of quantum projection measurements.

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To start with, let us consider that quantum state of a system is described by a density operator ρ, while τ is the evolution parameter responsible for ordering of physical events. For each value of τ, a family of projection operators is defined, which fulfil the following conditions:

$$
\begin{gathered}
M(\tau;\nu)M(\tau;\nu') = \delta_{\nu\nu'}M(\tau;\nu) \\
\sum_{\nu} M(\tau;\nu) = I
\end{gathered}
\tag{1}
$$

In Equation (1), operators $M(\tau;\nu)$ represent the properties of the system responsible for unitary evolution. Contrary to the traditional evolution theory, we postulate that the quantum state of the system is determined by a randomly chosen projection of its previous state by means of one of the $M(\tau;\nu)$ operators, for each value of the evolution parameter τ:

$$
\text{Prob}\,(\tau;\nu) = \text{Tr}\,[M(\tau;\nu)\rho(\tau - d\tau)] \tag{2}
$$

where $\rho(\tau - d\tau)$ describes a quantum state at the former value of the evolution parameter. In other words, it can be said that the projection postulate is used to obtain a new state of the system.

On the basis of the considerations above, it can be implied that the evolution parameter τ is time. In order to simplify our considerations, let us assume discrete values of time $t_0 < t_1 < t_2 < \ldots\, t_n$ .... In this notation, the projection evolution leads to the recurrent equation describing quantum states of the system:

$$
\rho(t_{n+1}) = \frac{M(t_{n+1};\nu_{n+1})\rho(t_n)M(t_{n+1};\nu_{n+1})}{Tr[M(t_{n+1};\nu_{n+1})\rho(t_n)M(t_{n+1};\nu_{n+1})]} \tag{3}
$$

In Equation (3), $\rho(t_n)$ stays for the quantum state of the system at the time t_n, while the state of the system in the following moment of time t_{n+1} is chosen randomly with the probability given by Eq. (2), where ν_{n+1} takes the values from the range required by Eq. (1).

Equation (3) leads to the conclusions that the evolution of the quantum system may follow different paths, as the series of projections are chosen randomly in moments of time t_0, t_1, ... using Eqs. (3) and (2), the probability of the fact that the quantum state ρ_0 of the system would be determined in a specific way, can be calculated:

$$
\begin{aligned}
\text{Prob}(t = t_0;\nu_0,\nu_1,\nu_2 \ldots,\nu_n) = \text{Tr}[&M(t_n;\nu_n)M(t_{n-1};\nu_{n-1}) \ldots M(t_0;\nu_0)\rho_0 \\
&M(t_0;\nu_0) \ldots M(t_{n-1};\nu_{n-1})M(t_n;\nu_n)
\end{aligned}
\tag{4}
$$

The time evolution is defined herein as a measurement made by Nature. However, the measurements made in a laboratory have to be a part of the evolution operator. The major conclusion of the theory presented is that there is no difference between the evolution and measurement, so the reduction of the quantum states is a natural part of the projection evolution. In particular cases, the projection evolution operators can be given by unitary operators which can be interpreted as traditional evolution operators $U(t)$ generated by Hamiltonian of the system:

$$M(t;\nu) = U(t-t_0)M(t_0;\nu)U^+(t-t_0) \tag{5}$$

When $U(t)$ commutate with $M(t_0;\nu_0)$, projection evolution operators are time independent.

On the other hand, when the operators (5) are applied to Eq. (3), we get unitary evolution of Schrödinger's type:

$$\rho(t) = U(t-t_0)\frac{M(t_0;\nu_0)\rho_0 M(t_0;\nu_0)}{Tr\left[M(t_0;\nu_0)\rho_0 M(t_0;\nu_0)\right]}U^+(t-t_0) \tag{6}$$

which means that the evolution leads to the real known Schrödinger equation. This leads to the conclusion that the unitary evolution and measurements can be independently considered as parts of the projection evolution.

2. Two-slit experiment

The two-slit experiment, known as the Young experiment, is schematically presented in Fig.1. In the most simplified version, the system consists of two slits, particles which are able to go through the slits and a screen.

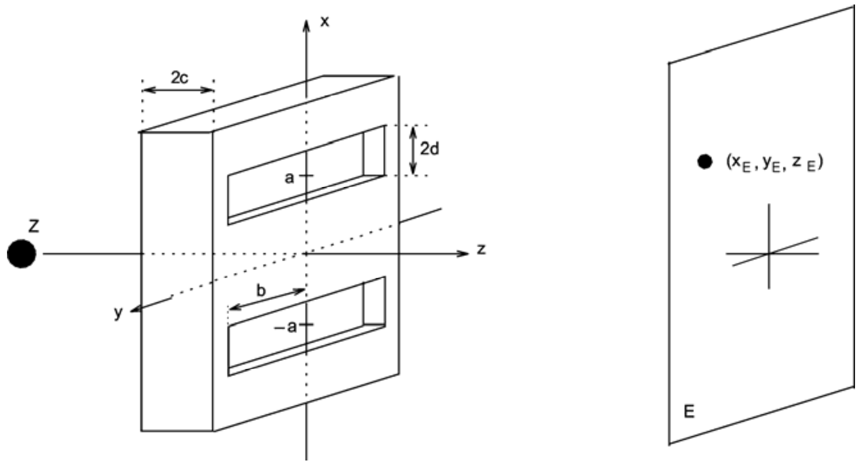

Fig. 1. Two-slit experiment [2]: Z – source, E – screen

The complementary rule is manifested in this experiment, showing the result of interaction between the system and the detector as the disturbance in a characteristic interference pattern on the screen. According to some attempts of interpretation of this event in the field of quantum physics, this interference pattern is destroyed as a result of the influence of a detector on the quantum state of the system in which the measurement is done. Quantum physicists claim that the act of observation causes significant changes in the quantum state of the system observed. In this paper, the projection evolution will be used in order to explain the influence of the detector on the quantum state of the system observed.

The main assumption in this approach is that the momentum broadening occurs for the particles which can be found between the source and the slits, and the slits and screen as well. This momentum broadening is represented by $\alpha_v(k)$ in the formalism describing the projection evolution.

The aim of the latter consideration is to determine the distribution value of probability that the particle would be found on the screen in a defined point $\vec{x}_E$. First of all, the evolution operator describing evolution of a particle which passes through the slits in its way from the source to the screen has to be constructed:

$$M(t;\nu)=\begin{cases} U(t-t_0)M_k(\vec{k})U^+(t-t_0) & \text{when} \quad t_0\le t<t_1;\nu=\vec{k}\in R^3\\ M_s(\nu) & \text{when} \quad t_1\le t<t_2;\nu=0,1\\ U(t_E-t_2)M_s(\nu)U^+(t_E-t_2) & \text{when} \quad t_2\le t<t_E;\nu=\vec{k}\in R^3\\ M_E(\nu') & \text{when} \quad t\ge t_E;\nu=0\,\text{lub}\quad \nu=\vec{x}_E\in X_E \end{cases} \tag{7}$$

where $M_k(\vec{k})=|\nu\rangle\langle\nu|$ projects on the states of the given momentum between the source an the slits:

- $M_s=\int_\Delta d^3x|\vec{x}\rangle\langle\vec{x}|$ projects on a slit state space.
- $M_k(\vec{k}')=|\nu'\rangle\langle\nu'|$ projects on a momentum state space between the slits and the screen.
- $M_E(\nu')=|\vec{x}_E\rangle\langle\vec{x}_E|$ is connected with screen states.
- $|\nu\rangle=|\psi_\nu\rangle=\int_{R^3}d^3\vec{k}\alpha_\nu(\vec{k})\left|\vec{k}\right\rangle$ where $\alpha_\nu(\vec{k})$ determines the shape of the wave packet.

The main point of interest is to know the position on the screen of the particle after passing the slits. The probability that the particle would be found on the screen in a point $\vec{x}_E$ is given by:

$$\begin{aligned}\text{Prob}(\vec{x}_E;\nu';\nu)=\text{Tr}[&M_E(\vec{x}_E)U(t_E-t_2)M_s(\nu)M_s(\Delta)U(t_1)M_\nu(\nu)\rho_0\\ &M_\nu(\nu)U^+(t_1)M_s(\Delta)M_s(\nu)U^+(t_E-t_2)M_E(\vec{x}_E)]\end{aligned} \tag{8}$$

Using Eqs. (7) and (8), the probability can be established by formula:

$$\text{Prob}(\vec{x}_E;\nu';\nu)=|\langle\nu|\psi_0\rangle|^2\left|\int_\Delta d\vec{x}\langle\vec{x}_E|U(t_E-t_2)|\vec{x}\rangle\langle\vec{x}|U(t_1)|\nu\rangle\right|^2 \tag{9}$$

It is assumed that the wave packet is broadened around given $\vec{k}_0$ in k_x, k_y, k_z axes directions and it has the shape of the Gaussian packet:

$$\alpha_\nu(k) = \frac{1}{(2\pi)^{3/2}\sqrt{\sigma\eta\mu}} \exp\left\{\frac{-(k_x - k_{0x})^2}{2\sigma}\right\} \exp\left\{\frac{-(k_y - k_{0y})^2}{2\eta}\right\} \exp\left\{\frac{-(k_z - k_{0z})^2}{2\mu}\right\} \quad (10)$$

where σ, η, μ are responsible for the shape of the wave packet in the directions of particular axes.

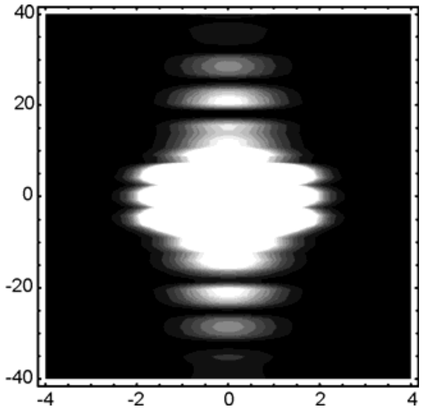

Fig. 2. The distribution value of probability on the screen for symmetric Gauss wave packet

As shown in Fig. 2, a lot of similarities can be found between the distribution value of probability obtained on the basis of theoretical analysis and the interference pattern observed in the experiment.

3. Influence of the detector

Another aspect of the two-slit experiment which needs to be considered is the influence of the detector on the quantum state of the system being under observation. In particular, one question should be answered: what will happen if we put a detector in one of the slits and this way disturb a quantum state of the system. That kind of situation implies the modification of the evolution operator (7) by the detector operator represented by:

$$M_D(\kappa) = \begin{cases} M_D(u) = \int_{\Delta'_u} d^3\vec{x}\,|\vec{x}\rangle\langle\vec{x}| & \text{if particle passes through the upper slit, } \kappa = u \\ M_D(d) = \int_{\Delta'_d} d^3\vec{x}\,|\vec{x}\rangle\langle\vec{x}| & \text{if particle passes through the lower slit, } \kappa = d \\ I - M_D(u) - M_D(d) & \text{if particle hits the wall, } \kappa = w \end{cases} \quad (11)$$

In the result of that modification, the projection evolution operator will be described by the following expressions:

$$M(t;\nu)=\begin{cases} U(t-t_0)M_k(\vec{k})U^+(t-t_0) & \text{when} \quad t_0 \le t < t_1; \nu=\vec{k}\in R^3, \\ M_s(\nu) & \text{when} \quad t_1 \le t < t_2; \nu = 0,1, \\ M_D(\kappa) & \text{when} \quad t_2 \le t < t_3; \nu=\kappa=u,d,w, \\ U(t_E-t_2)M_s(\nu)U^+(t_E-t_2) & \text{when} \quad t_2 \le t < t_E; \nu=\vec{k}\in R^3, \\ M_E(\nu') & \text{when} \quad t \ge t_E; \nu = 0 \, or \quad \nu=\vec{x}_E \in X_{E.} \end{cases} \tag{12}$$

Using this form of the projection evolution operator (13) one can calculate the probability of finding a particle on a screen:

$$\text{Prob}(\vec{x}_E.k',k)=\left|\langle x_E|U(t_E-t_3)|\nu'\rangle\langle\nu'|M_s(\Delta)M_D(\kappa)U(t-t_1)|\nu\rangle\langle\nu|\psi_0\rangle\right|^2 \tag{13}$$

where $\kappa = u$ describes the particle passing through the upper slit and $\kappa = d$ describes the particle passing through the lower slit. In both cases, the distribution of probability on the screen is the same as for the diffraction pattern of one slit.

4. Conclusions

In many theories describing quantum physics the concept of measurement is related to the reduction of the packet wave, which is explained by the influence of an observer or a detector on system observed. It is not necessary to involve the idea of influence of a detector on a quantum state in projection evolution because it comes from the main assumption of the theory: time evolution is defined as a measurement made by Nature in every stage of the experiment. As described above, the influence of an observer on the system is natural, and comes from the theory, not from different assumptions.

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References

[1] GRABOWSKI M., INGARDEN R.S., *Quantum Mechanics*, Warszawa, PWN, 1989 (in Polish).
[2] GÓŹDŹ A., DĘBICKI M., PIETROW M., Int. J. Mod. Phys., E14 (2005), 477.
[3] PRESKILL J., *Quantum Information and Computation*, Lecture Notes for Physics 229, California Institute of Technology, September, 1998.

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